PEAKS2TAILS

Deep Quant Finance

1) Primer

1.1. Math Primer

The module contains essential math toolbox for Quant Finance. These topics are foundational for any Quant finance use case e.g. pricing, risk modeling, trading, portfolio optimization etc. The following topics will be covered. All concepts are covered with derivation of key formulas and dynamic visualizations in Excel.

- Set Theory Basics
- Relations & Functions
- Complex Numbers
- Limits and Derivatives
- Taylor Series
- Integration
- Differential Equations
- Vectors
- Matrices
- Partial Derivatives
- Convex Optimization
- Numerical Integration
- Elementary Probability Theory
- Measure Theory (Filtration, $\sigma - algebra$)
- Named Distributions (Discrete)
- Named Distributions (Continuous)
- Joint distributions
- Marginal and Conditional densities
- Conditional Expectation
- Linear Regression
- Time Series Analysis
- Fourier Analysis

Assignment	▶ **Exercise Set** ▶ **Question Set on Probability and Expectation for Interviews**

1.2. Python Primer

Learn Python basics to perform routine tasks such as arithmetic operations, manipulating data, visualizing data and writing functions and custom classes for various Quant Finance use cases such as pricing and risk modeling. The topics are as follows

- Data Structures in Python
- If statements & loops
- Functions and Lambdas

- Object oriented programming

You will also learn some of the popular python modules such as

- NumPy – Vectorized calculations
- Pandas – 2D data reading, writing, cleaning
- Matplotlib – Basic visualization
- Seaborn and Plotly – for advanced visualization
- SciPy –probability, statistics and optimization
- Statsmodels – Regression and Timeseries analysis
- Scikit-Learn – Machine learning algorithms

Assignment	► **Problem Set on individual topics**
Project	► **Create a custom class for Black-Scholes to return price of European call and put options and option Greeks** ► **Implement numerical integration class to calculate CDF under normal distribution using Trapezoidal rule and Simpson's 1/3rd rule** ► **Fit timeseries ARMA model on GDP and unemployment rates and perform stationarity tests**

1.3. Stochastic Calculus

In this chapter, we will do a deep dive into stochastic calculus which is an indispensable tool to model random behavior of assets and come up with pricing formulas for derivatives. We will learn about Brownian motion, stochastic processes such as GBM, OU processes and important properties such as Markov and Martingales. We will do a deep dive into Ito Calculus and solve some key stochastic differential equations using Ito integral. We will also learn some important theorems such as Girsanov theorem and change of measure technique which are essential tools for derivative pricing. The list of topics is as follows

- Brownian Motion and Properties
- Stochastic Processes
- Parameter estimation
- Markov vs Martingale Properties
- Ito Calculus
- Radon Nikodym Derivative
- Girsanov Theorem
- Change of Measure
- Basic Monte Carlo for Simulation

Assignment	► **Problems on properties of Ito's Lemma, Ito isometry and Ito Integral**

Project	▶ **Create custom library for simulating stochastic processes (ABM, GBM & Ornstein-Uhlenbeck process)**

1.4. Markov Models

In this chapter, we will learn about Markov states and transition probabilities. Markov models have profound applications in the areas of credit risk, Insurance, quant finance and machine learning algos such as reinforcement learning. The topics in this module are as follows

- Markov Chains
- Time Homogeneous – 2 State Model
- Time Homogeneous – Multi State Model
- Kolmogorov forward and backward equations
- Solving state transition probabilities
- Time inhomogeneous Markov

Assignment	▶ **Solving Kolmogorov Equations to obtain transition probabilities**
Project	▶ **Loss distribution of a Credit Portfolio using transition matrix**

2) Portfolio Management

2.1. Mean Variance Optimization

We learn about the contributions of Harry Markowitz (the father of modern portfolio theory). We will discuss the mean variance utility functions; risk return trade off. We will construct the efficient frontier, tangency portfolio and global minimum variance portfolio. Then we will solve a few optimization problems using constraints on mean return and variance of the portfolio using Lagrangian multiplier. The topics are as follows

- Modern Portfolio Theory
- Efficient Frontier
- Global Minimum Variance Portfolio
- Sharpe Ratio
- Capital Market Line (CML)
- Tangency Portfolio
- Convex Optimization with return constraints

Project	**Calculate optimal allocation of a portfolio of 4 assets given mean and covariance data. Plot the optimal portfolio on the efficient frontier**

2.2. CAPM & Factor Models

In this chapter, we explore some popular factor models i.e. one factor models such as CAPM and then multi factor models such as Fama-French model. We will learn about the rationale behind factors such as value-growth, size, momentum. We will test the significance of the factor models using historical data. The topics are as follows

- CAPM Model
- Systematic Risk vs Idiosyncratic Risk
- Security Market Line (SML)
- Factor Investing
- Fama-French model

Case Study	**Evaluate performance of factor models R^2 and test the significance of alpha and beta values on historical data**

2.3. Black-Litterman model

The Black-Litterman model is a Bayesian approach that enables investors to incorporate their views regarding the performance of various assets. The components of the model are as follows

- Bayes formula – likelihood, prior and posterior
- Reverse optimization to get prior
- Incorporating Views
- Obtain Posterior distribution
- Asset Allocation

We will perform one complete numerical example with relative and absolute views of the investor and compute optimal asset allocation mix.

2.4. Active Portfolio Management

This chapter deals with traditional active portfolio management techniques which are massively popular in industry today. We will learn about large scale portfolio constrained optimization problems surrounding alpha, beta, tracking error, concentration, turnover etc. We will learn about hard constraints, soft constraints, eigen value decomposition to ensure covariance matrices are positive semi definite to allow for convex optimization.

Case Study	▶ **Construct a real-life model portfolio optimization**

2.5. Robust Optimization

This chapter deals with robust estimation of parameters in the presence of uncertainty. The topics are as follows

- Practical problems with mean variance optimization
- Robust statistics
- Robust estimation of Regression parameters
- Shrinkage estimation
- Robust resampling techniques
- Robust Optimization

Case Study	• **Compare the performance of portfolio using Robust optimization vs Traditional Optimization**

2.6. Stochastic control

Formulate and solve a dynamic portfolio optimization problem using stochastic control combining views with market data. Understand the importance of behavioral biases and remove them. Implement a dynamic portfolio optimization model. Some of the important topics we will cover in this chapter are as follows

- Dynamic Programming
- Stochastic Control
- Kelly Criterion
- Martingale Duality
- Risk Sensitive Control
- Kalman Filter

Case Study	• **End to end implementation of dynamic portfolio optimization using stochastic control**

2.7. Statistical Arbitrage (Pairs trading)

Long-short Pairs trading strategy is a classic statistical arbitrage trading strategy where the buy and sell decision are based on the mean reverting spread between 2 co-integrated assets. In this chapter we will learn the following topics

- Co-integration
- OU-fitting
- Design a long-short trading strategy
- Backtest

Case Study	**Identify two co-integrated stocks and implement end to end pairs trading strategy including performance Backtesting**

3) Equity Derivatives

3.1. Binomial Tree Model

In this chapter, we will learn about pricing equity derivatives using Binomial Tree model. The topics we will learn are as follows

- Multi-step binomial tree
- Replicating Portfolio Approach
- Risk Neutral (Martingale) Measure
- Vanila European options
- Lookback option
- Barrier option
- Stopping times & American Option

Assignment	**Question set on Binomial Tree based pricing**
Project	**Custom class for creating Binomial tree-based option pricing engine for European options**

3.2. Black-Scholes Equation

A deep dive into the Black-Scholes model for Option pricing. We will learn the following

- Black-Scholes PDE from delta hedging
- Generalized Black-Scholes Formula
- BSM price for European Call and Put
- Put Call parity
- BSM Greeks
- Feynman-Kac theorem
- Green Functions
- Implied volatility
- Volatility variations

Assignment	**Exercise Set**

3.3. Finite Difference Methods

Given the pricing PDE, one can price a range of options using a method called Finite difference grids. In this chapter, we will learn various finite difference schemes, their accuracy & stability and price a few derivatives such as Vanilla and Asian options. The concepts we will learn are as follows

- Numerical sensitivities
- Explicit schemes
- Implicit schemes
- Stability Analysis
- Crank-Nicolson
- Douglas schemes
- Richardson extrapolation
- ADI and Hopscotch methods

Project	**Pricing a Bermudan Option using Finite difference schemes**

3.4. Local Volatility Models

In this chapter, we will relax the assumption that volatility is constant. We will make the volatility level and time dependent. The topics we will cover are as follows

- Dupire Local volatility
- Alternate asset pricing models
- Market Implied volatility smile

- Variance swaps
- Implied volatility representation of local volatility
- Arbitrage free conditions for option prices
- Advanced implied volatility interpolation
- Simulation of local volatility

Assignment	▶ **Exercise Set**

3.5. Jump Process

In this chapter, we will incorporate jumps into the dynamics of the asset prices. The topics we will cover are as follows

- Ito's lemma and jumps
- Partial Integro differential equations for jump diffusion
- Analytical option prices
- Characteristic function of Merton's Model
- Dynamic hedging of jumps with BSM
- Finite Exponential Levy process
- CGMY and Variance Gamma process
- CGMY process

Assignment	▶ **Exercise Set**

3.6. COS method for European Option Valuation

In this chapter we explore the Fourier cosine series for option valuation.

- Density approximation via Fourier cosine approximation
- Pricing European options using COS method
- Error Analysis

Assignment	▶ **Exercise Set**
Case Study	▶ **Numerical COS method results for GBM, CGMY & VG process**

3.7. Change of Measure in Multi-dimensions

In this chapter, we will explore the change of measure technique for a vector of stochastic processes. The topics we will cover include

- Multidimensional SDE systems
- Cholesky Decomposition

- Ito's lemma for vector processes
- From $\mathbb{P}$ to $\mathbb{Q}$ in BSM
- Affine diffusion processes
- Affine Jump diffusion processes

Assignment	**Exercise Set**

3.8. Stochastic Volatility Models

In this chapter, we will treat volatility as a separate stochastic process. The topics we will cover include

- SABR and Heston stochastic volatility models
- Pricing PDE
- Model Calibration
- Heston characteristic function
- COS method for Heston model
- Heston model with piecewise constant parameters
- Bates model

Assignment	**Exercise Set**
Case Study	**Parameter study for implied volatility skew and smile**

3.9. Monte Carlo Simulation

In this chapter, we will explore the Monte Carlo methods for pricing derivatives. The topics we will cover include

- Monte Carlo Integration
- Euler-Maruyama scheme
- Milstein scheme
- Explicit Scheme
- Quadratic Exponential Scheme
- Antithetic Sampling
- Control Variate method
- Low discrepancy Sequence
- Pathwise sensitivities
- Likelihood ratio methods

Assignment	▶ **Exercise Set**
Project	▶ **Monte Carlo option pricing for Asian, Digital and Lookback options**

4) Interest Rate & FX Derivatives

4.1. Short rate models

In this chapter, we will learn the popular short rate models and pricing of options on zero coupon bonds

- Spot rate, forward rate
- Vasicek Model
- CIR model
- HJM Framework
- Short rate dynamics under HJM Framework
- Solution to Hull and White model
- Option on zero coupon bonds under Hull and White

Assignment	▶ **Exercise Set**
Case Study	▶ **Hull & White calibration of parameters**

4.2. Interest rate and FX derivatives

In this chapter we will learn to value popular interest rate derivatives

- FRAs
- Swaps
- Caps/Floors
- Swaptions
- Interest rate Parity
- FX Forward, swaps and xCCy
- FX Options
- Modeling Correlation between IR and FX rates

Assignment	▶ Exercise Set

4.3. RFR – the new regime

With LIBOR transition to RFR, we need to update out understanding in terms of changes to yield curve construction, valuation of old and new rate products, sensitivity and risk calculations and hedging strategies.

- History of LIBOR Transition
- Difference between LIBOR and RFR
- Fall Back rates
- SOFR proxy by means of OU process
- SOFR term rate calculation process
- Strips and Rolls
- Valuation of SOFR Futures and swaps
- Valuation of SOFR future options
- Single Currency Curve Construction
    - Market data & Knots
    - Interpolation styles (log-linear, log-cubic)
    - Numerical Solver
    - Pricing biases and Risk Considerations
    - Impact of CCP basis on curve construction
- Multi-Currency Curve Modeling
    - Calibration to XCS, FX Swap
    - Interdependence between basis, DF and forward FX rates
    - Global (Simultaneous) calibration
- Sensitivity and Risk
    - Analytical computation
    - Tangent Mode
    - Adjoint Mode (AAD)
    - Curve Jacobians
    - Ultra-fast curve building
    - Modeling Jumps, Spikes and Turn of the Year effect
    - Delta, gamma and cross gamma risks
    - Swaption pricing and Greeks
    - Modeling Volatility Surface
    - Forward volatility and correlations
    - Value at Risk and Expected Shortfall calculations
- Hedging
    - Duration Matching
    - PCA hedge
    - CoVaR hedge
    - Example – Hedging SOFR term rate with SOFR futures via jump process
    - Example – Hedging Caps and Floors with SOFR future options

Assignment	▶ Exercise Set
Project	▶ Develop a Python library with swaps, curves and schedules to implement a curve solving algorithm. Implement Gradient-descent, Gauss-Newton and Levenberg-Marquardt methods. Calculate sensitivities using automatic differentiation

5) Credit Derivatives

In this chapter, we will explore the world of credit derivatives. The topics we will cover include

- Structural models
- Intensity models
- Correlated default times using Copula
- Valuation of CDS, CLN and Basket products
- Hazard rate calibration
- Jarrow-Lando-Turnbull

Assignment	▶ Exercise Set
Project	▶ Price a fair spread for a portfolio of CDS for a basket CDS

6) AI in risk and finance

In this chapter we will learn about the state-of-the art AI and ML techniques being used today in finance and risk management. The topics include

- Neural Network for Option Pricing
- LSTM for stock price prediction
- Random forest and XGBoost for accurate return and volatility forecasting portfolio optimization
- Large Language models in Quant Finance
- Genetic AI for portfolio selection
- Reinforcement learning for portfolio balancing
- CNNs for volatility surface modeling